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GENERAL THEORY OF MICROLOG

Abstract

In this paper there are formulas determined for calculation of the electrode constants of Microlog for various shape geometry of electrodes. It is possible to compare fields of relations remarked as (k/a) = f (2m/a A/a) for various shapes of electrodes. Curvilinear segments of curves are observed for short distances between electrodes and their curvature is influenced with the magnification factor remarked like A/a. For long distances between electrodes we observe linearization of curves that depends on the translation factor only being remarked as 2m/a. This is translation factor in horizontal direction. There exist also the translation factor in vertical direction remarked as 2n/a, because relation for constant in general can be written like k/a = f (2m/a, 2n/a, A/a). Linear segments of curves are allowed to use the equation for the point electrodes, because error is in such case negligible.

In electrically-homogeneous surroundings there is registered identical resistivity. The curves of the micro-normal and the micro-inverse must have the same resistivity value. This is insured with condition that it holds that kM = 2×kN. The mentioned condition is generally valid both for linear and nonlinear relations.

Electrode systems of Microlog can be symmetrical and asymmetrical. Asymmetrical systems have the right asymmetry or the left asymmetry. It is expressed with the help of factor ε. This can be positive, negative or zero.

The paper makes possible form arbitrary electrode arrays. They can be three-electrodes or four-electrodes. Their electrodes are allowed to assume an arbitrary position on plane. What is important is you can exactly compute for any array constants characterizing this array.

Key words: electrode constant, the micro-normal, the micro-inverse, linear segments, nonlinear segments, magnification factor, translation factor, the point electrodes, resistivity, right asymmetry, left asymmetry, Microlog, well-logging.

Introduction

In several former works there was stated the fact that geometry of electrodes, namely their shape, effects their electric field, thus their functions k/a = f (2m/a, 2n/a, A/a) have outstanding differences between themselves. It holds mainly for near distances of electrodes. For long distances influence of shape geometry and dimensions of electrodes are falling down and there survives only the influence of electrode spacing.

The electric field tends to field of the point electrodes and just this is able to make calculation easier. Therefore there are preferred those curve segments having function k/a = f (2m/a, A/a) linear. However, it does not take as a rule.

The next problem is definition of constants for the micro-normal and the micro-inverse one to other. If I have to register identical resistivity for electrically-homogeneous surroundings, the both constants should bee equal. However, in electrically-inhomogeneous surroundings there are observed various resistivity of the

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micro-normal and the micro-inverse, because horizontal radius of both arrays is different and the resistivity is changed in horizontal direction.

Both mentioned problems are solved in this paper together with explanation of symmetry and asymmetry of used arrays of electrodes.

Comparison of shape geometry and dimensions of electrodes

In fig.1 there are depicted various systems of Microlog for electrodes of various geometry. Each of them consists of the current electrode A and two potential electrodes N and M. There is registered the micro-normal being on electrode M, more distant electrode, and the micro-inverse between electrodes M and N.

I distinguish disc electrodes and square electrodes. I can compare the square electrodes in two variances. The first has the square electrodes situated on one of sides of square; I signed this system like the square electrodes. The second has the same electrodes situated on one of vertexes of square – I remarked such system like the diamond electrodes, because it is special case of diamond.

The fundamental functional relation has this form; k/a = f (2m/a, 2n/a, A/a). The ratio remarked as (2m/a) presents translation factor in horizontal direction, the ratio (2n/a) is translation factor in vertical direction and the ratio remarked as (A/a) is magnification factor. If it holds that (2n/a) = 0, you will receive simpler form: k/a = f (2m/a, A/a). This relation is depicted in fig.2 for all three various shapes of electrodes. For near distances when it holds that (2m/a) → 1 there exist an influence of electrode dimensions. For various (A/a) I observe various shapes of the depicted curves, as well. However, for long distances when (2m/a) >> 1 the relationship inclines to the line for the point electrodes defined with the help of this formula:

$$\left(\frac{k}{a}\right) = 2\pi \times \left(\frac{2m}{a}\right). \tag{1}$$

This relation is marked with dash-and-dot line. And you can see that for (2m/a) = 1 you will get that (k/a) = 2π.

From point of view of relation k/a = f (2m/a, 2n/a, A/a) the disc electrodes provide for calculation of the micro-normal the following formulas:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\sum_{i=1}^{4} G_i}, \tag{2}$$

$$G_1 = +\frac{1}{4} \times \left(\frac{A}{a}\right) \times \left\{ \left(\frac{2n}{a} + \frac{A}{a} + 1\right) \times \mathrm{Argsinh}\left[\left(\frac{2m}{a}\right) \times \left(\frac{2n}{a} + \frac{A}{a} + 1\right)\right] - \left(\frac{2n}{a} + \frac{A}{a} - 1\right) \times \mathrm{Argsinh}\left[\left(\frac{2m}{a}\right) \times \left(\frac{2n}{a} + \frac{A}{a} - 1\right)\right] \right\}, \tag{3}$$

$$G_2 = -\frac{1}{4} \times \left(\frac{A}{a}\right) \times \left\{ \left(\frac{2n}{a} - \frac{A}{a} + 1\right) \times \mathrm{Argsinh}\left[\left(\frac{2m}{a}\right) \times \left(\frac{2n}{a} - \frac{A}{a} + 1\right)\right] - \left(\frac{2n}{a} - \frac{A}{a} - 1\right) \times \mathrm{Argsinh}\left[\left(\frac{2m}{a}\right) \times \left(\frac{2n}{a} - \frac{A}{a} - 1\right)\right] \right\}, \tag{4}$$

and

$$G_3 = -\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{2n}{a} + \frac{A}{a} + 1\right)^2} - \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{2n}{a} + \frac{A}{a} - 1\right)^2} \right\}, \tag{5}$$

$$G_4 = +\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{2n}{a} - \frac{A}{a} + 1\right)^2} - \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{2n}{a} - \frac{A}{a} - 1\right)^2} \right\}. \tag{6}$$

If it holds that (2n/a) = 0 you attain simpler formulas:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\left(F_1 + F_2\right)}, \tag{7}$$

$$F_1 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \left(\frac{A}{a} + 1\right) \times \text{Argsinh}\left[\left(\frac{2m}{a}\right)^{-1} \times \left(\frac{A}{a} + 1\right) \right] - \left(\frac{A}{a} - 1\right) \times \text{Argsinh}\left[\left(\frac{2m}{a}\right)^{-1} \times \left(\frac{A}{a} - 1\right) \right] \right\}, \text{ and}$$

(8)

$$F_2 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{A}{a} + 1\right)^2} - \sqrt{\left(\frac{2m}{a}\right)^2 + \left(\frac{A}{a} - 1\right)^2} \right\}. \tag{9}$$

Equivalent formula is the formula derived for system expressed in polar coordinates. It is again relation for factors (2m/a), (2n/a) and (A/a).

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\left(F_1 + F_2\right)},$$

$$\left(\frac{2r}{a}\right) = \sqrt{\left(\frac{2n}{a}\right)^2 + \left(\frac{2m}{a}\right)^2}, \tag{10}$$

$$F_1 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left(\frac{2r}{a}\right)^{-1} \times \left(\frac{2}{\pi}\right) \times \left\{ \left(\frac{2r}{a} + \frac{A}{a}\right) \times \mathrm{E}\left\{ \left(\frac{2r}{a} + \frac{A}{a}\right)^{-1} \right\} - \left(\frac{2r}{a} - \frac{A}{a}\right) \times \mathrm{E}\left\{ \left(\frac{2r}{a} - \frac{A}{a}\right)^{-1} \right\} \right\}, \text{ and} \tag{11}$$

$$F_2 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left(\frac{2r}{a}\right)^{-1} \times \left\{ \arcsin\left(\frac{2r}{a} + \frac{A}{a}\right)^{-1} - \arcsin\left(\frac{2r}{a} - \frac{A}{a}\right)^{-1} \right\}. \tag{12}$$

If you implement again condition that (2n/a) = 0, you will get simpler formulas:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\left(F_1 + F_2\right)},$$

$$F_1 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left(\frac{2m}{a}\right)^{-1} \times \left(\frac{2}{\pi}\right) \times \left\{ \left(\frac{2m}{a} + \frac{A}{a}\right) \times \mathrm{E}\left\{ \left(\frac{2m}{a} + \frac{A}{a}\right)^{-1} \right\} - \left(\frac{2m}{a} - \frac{A}{a}\right) \times \mathrm{E}\left\{ \left(\frac{2m}{a} - \frac{A}{a}\right)^{-1} \right\} \right\}, \text{ and} \tag{13}$$

$$F_2 = +\frac{1}{2} \times \left(\frac{A}{a}\right)^{-1} \times \left(\frac{2m}{a}\right)^{-1} \times \left\{ \arcsin\left(\frac{2m}{a} + \frac{A}{a}\right)^{-1} - \arcsin\left(\frac{2m}{a} - \frac{A}{a}\right)^{-1} \right\}. \tag{14}$$

where $\mathrm{E}\left\{ \left(\frac{2m}{a} \pm \frac{A}{a}\right)^{-1} \right\}$ = the complete elliptic integral of second type, m = the distance between both centres of the potential and current electrodes in horizontal direction [m],

n = the distance between both centres of the potential and current electrodes in vertical direction [m],

A = the diameter of the current electrode [m], and

a = the diameter of the potential electrode [m].

Both used formulas yield identical results.

You ought to note that an influence of factor (A/a) for the disc electrodes is the lowest of all three depicted systems. For (2m/a) > 5 there exist function k/a = f (2m/a, A/a) like linear and independent on the ratio (A/a) = 1. If (A/a) = 5, all linear relation would be only for (2m/a) > 13. All this is well visible in fig.2.

The square electrodes have more significant influence of the electrode dimensions – it is distinct in fig.2, too. The disc electrodes you can easy replace by the point electrodes and those constants can be enumerated after simple formula (1) – it is main advantage in comparison to the square electrodes.

For relationship of the square electrodes k/a = f (2m/a, 2n/a, A/a) there are valid the formulas as follows:

$$\left(\frac{k}{a}\right)=\frac{2\pi}{\sum_{i=1}^{4} G_i},$$

$$G_1=+\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\frac{2n}{a}+\left(\frac{A}{a}+1\right)\right]\times\operatorname{Argsinh}\left\{\frac{\frac{2n}{a}+\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}-\left[\frac{2n}{a}+\left(\frac{A}{a}-1\right)\right]\times\operatorname{Argsinh}\left\{\frac{\frac{2n}{a}+\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}\right\}, \tag{15}$$

$$G_2=+\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\frac{2n}{a}-\left(\frac{A}{a}+1\right)\right]\times\operatorname{Argsinh}\left\{\frac{\frac{2n}{a}-\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}-\left[\frac{2n}{a}-\left(\frac{A}{a}-1\right)\right]\times\operatorname{Argsinh}\left\{\frac{\frac{2n}{a}-\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}\right\}, \tag{16}$$

$$G_3=-\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}+\left(\frac{A}{a}+1\right)\right]^2}-\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}+\left(\frac{A}{a}-1\right)\right]^2}\right\}, \text{ and} \tag{17}$$

$$G_4=-\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}-\left(\frac{A}{a}+1\right)\right]^2}-\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}-\left(\frac{A}{a}-1\right)\right]^2}\right\}. \tag{18}$$

If it holds that (2n/a) = 0 you will attain function k/a = f (2m/a, A/a) and then you will get simpler formulas:

$$\left(\frac{k}{a}\right)=\frac{2\pi}{\left(F_1+F_2\right)},$$

$$F_1=+\frac{1}{2}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left(\frac{A}{a}+1\right)\times\operatorname{Argsinh}\left\{\frac{\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}-\left(\frac{A}{a}-1\right)\times\operatorname{Argsinh}\left\{\frac{\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}\right\}, \text{ and} \tag{19}$$

$$F_2 = -\frac{1}{2}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left(\frac{A}{a}+1\right)^2}-\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left(\frac{A}{a}-1\right)^2}\right\}. \tag{20}$$

An influence of the electrode dimensions is distinctly significant. I think this case is biggest of all three systems when the electrode dimensions have highest affect on constant. Linear segment for (A/a) < 2 you can observe only if (2m/a) > 40. Nonlinear segments of function can be used for calculation too; however, the distances between electrodes will not be identical like it is for the point electrodes.

The micro-normal of the diamond electrodes for relation k/a = f (2m/a, 2n/a, A/a) is presented by these formulas:

$$\left(\frac{k}{a}\right)=\frac{2\pi}{\sum_{i=1}^{4} G_i},$$

$$G_1 = +\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]\times\text{Argsinh}\left\{\frac{\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}-\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]\times\text{Argsinh}\left\{\frac{\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}\right\}, \tag{21}$$

$$G_2 = +\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]\times\text{Argsinh}\left\{\frac{\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}-\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]\times\text{Argsinh}\left\{\frac{\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}\right\}, \tag{22}$$

$$G_3 = -\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]^2}-\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2}\right\}, \text{ and} \tag{23}$$

$$G_4 = -\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]^2}-\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2}\right\}. \tag{24}$$

If you implement condition that (2n/a) = 0 you will obtain simpler formulas:

$$\left(\frac{k}{a}\right)=\frac{2\pi}{\left(F_1+F_2\right)},$$

$$F_1 = +\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]\times\text{Argsinh}\left\{\frac{\sqrt{2}\times\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}-\left[\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]\times\text{Argsinh}\left\{\frac{\sqrt{2}\times\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}\right\}, \text{ and} \tag{25}$$

$$F_2 = -\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \sqrt{\left[\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1\right)\right]^2 + 2 \times \left(\frac{A}{a} + 1\right)^2} - \sqrt{\left[\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1\right)\right]^2 + 2 \times \left(\frac{A}{a} - 1\right)^2} \right\}. \tag{26}$$

Affect of the electrode dimensions is strong and it is visible on relations k/a = f (2m/a, A/a) depicted in fig.2. Nevertheless, for (A/a) = 1 the linear segment will present by (2m/a) > 4 yet. This is momentous exception being comparable to the disc electrodes. The only mentioned exception presents relation which can be replaced by relation for the point electrodes with negligible error.

For remaining ratios (A/a) there are no any exceptions; for example, when (A/a) =2 there will begin the function to be linear only if (2m/a) > 40 and for the next ratios the limits will be even higher.

Calculation of constants for the micro-normal and the micro-inverse

For the axis arrays if it is the micro-potential registered on the electrode M being the more distant one there will hold this equation:

$$U_M = \frac{1}{k_M} \times R \times I \ , \tag{27}$$

where I = the current flowing through electrode A [mA], and
R = the resistivity of surroundings [Ωm].

For the micro-potential registered on electrode N being the closer one you can use the similar relation:

$$U_N = \frac{1}{k_N} \times R \times I \ . \tag{28}$$

If you want to register the micro-inverse, you will have to register the difference of two potential; UM and UN.

$$\Delta U = U_N - U_M \ . \tag{29}$$

After substitution of equations (27) and (28) into equation (29) you obtain the following expression:

$$\Delta U = \left(\frac{1}{k_N} - \frac{1}{k_M}\right) \times R \times I \ . \tag{30}$$

Now, I express the resistivity R and I shall receive this formula:

$$R = \left(\frac{1}{k_N} - \frac{1}{k_M} \right)^{-1} \times \frac{\Delta U}{I} \,. \tag{31}$$

Further, you are able to write down the formulas expressing constants of the micro-normal and the micro-inverse.

$$k_p = k_M \,, \quad \text{and} \tag{32}$$

$$k_g = \left(\frac{1}{k_N} - \frac{1}{k_M} \right)^{-1} , \tag{33}$$

where kp = the constant of the micro-normal [m],
kg = the constant of the micro-inverse [m],
kM = the constant of the micro-normal being on electrode M [m], and
kN = the constant of the micro-normal being on electrode N [m].

Relations being between constants of the micro-normal and the micro-inverse

In the electrically-homogeneous surroundings I must receive identical resistivity. Therefore it is possible to use these relations:

$$R = k_p \times \frac{U_M}{I} \,, \quad \text{and} \tag{34}$$

$$R = k_g \times \frac{\Delta U}{I} \,. \tag{35}$$

Both left sides of equations are same and this is reason why the right sides must be identical too. There must it be held that:

$$\frac{\Delta U}{I} = \frac{U_M}{I} \,, \quad \text{and} \tag{36}$$

$$k_g = k_p = k_m \,. \tag{37}$$

To realize conditions (36) and (37) – it means to implement this condition:

$$\left(\frac{k_M}{a} \right) = 2 \times \left(\frac{k_N}{a} \right). \tag{38}$$

This condition through formula (33) for kg is able to ensure relation (37). And simultaneously through the ratio that

$$\frac{U_N}{U_M} = 2\,,$$

resulting from formulas (27) and (28) and further through equation (29) it is confirmed that relation (36) is valid.

Equation (38) holds for linear and nonlinear relations generally. However, for linear segments of relations it is valid that condition (38) is easy realizable, because in domain of linear relations there holds that:

$$\left(\frac{2m}{a}\right)_M = 2\times\left(\frac{2m}{a}\right)_N. \tag{39}$$

This equation results from equation for the point electrodes:

$$k = 4\pi \times m\,. \tag{40}$$

Linear segments of relations are for the disc electrodes and the only linear segment existed also for the diamond electrodes. For example, the disc electrodes have linear relation for (A/a) = 1 if it holds that (2m/a) > 5; the diamond electrodes for (A/a) = 1 have linear relation for (2m/a) > 4. The resting relations have nonlinear; or else condition (38) holds, but distances, AM and AN, are not double – equation (39) is not valid.

We observe for linear relation symmetry, AN = MN, whereas, for nonlinear relation there is asymmetry, AN ≠ MN. This is visible difference between linear and nonlinear relations.

Comparison of exact calculations of constants with experimental modeling

For comparison there were used the disc electrodes of Micrologʼ manufactured in Russia. Such system is characterized with following characteristics: a = 0.010m, AN = 0.025m, AM = 0.050m. Thanks to those characteristics it is possible to compute ratios (2m/a)M and (2m/a)N.

$$\left(\frac{2m}{a}\right)_N = \frac{2\times 0.025}{0.010} = 5,\ \text{and}\ \left(\frac{2m}{a}\right)_M = \frac{2\times 0.050}{0.010} = 10.$$

After these ratios you attain that (kN/a) = 31.821 and (kM/a) = 63.040. For a = 0.010m it proceeds that kN = 0.32m and kM = 0.63m. Ratio being between both constants is 1.969. It is very near to 2.

If you suppose the point electrodes, you will use formula (1) for values (2m/a)N = 5 and (2m/a)M = 10. Then you will get the values as follows:

$$\left(\frac{k_N}{a}\right) = 2\pi\times 5 = 31.416\ \text{and}\ \left(\frac{k_M}{a}\right) = 2\pi\times 10 = 62.832\,.$$

For a = 0.010m again you receive kN = 0.32m and kM = 0.63m. You have again identical values comparable with former calculation.

Now, it is possible to determine constants of Microlog after formulas (32) and (33). For the micro-normal it holds that kp = kM = 0.63m. For the micro-inverse I receive that kg = (0.32-1 – 0.63-1) -1 = 0.65m. The calculated constants are not fully identical, but, they are very close to one other. In the practice we observe often a small difference between resistivity of the micro-normal and the micro-inverse what depends on accuracy of calculation.

The next figures, fig.3 and fig.4, present the correction charts for Microlog of Russian production. They vary according to dimensions of the electrode pad. The pad having dimensions 100 × 200mm has kp = 0.48m and kg = 0.34m, whereas, the pad with dimensions 70 × 190mm presents kp = 0.56m and kg = 0.36m. It is more than clear there are big differences between values exactly calculated and empirically modelled.

Just that is why I have to remember that the values empirically modelled are influenced with the electrode potentials being on surface of electrodes and, too, by an influence of dimensions of the electrode pad. It is known that empirically modelled constants are lower due to both significant factors effecting registration.

Comparison of systems having different shape of electrodes

I shall attempt in this chapter to project distances between electrodes for different shape geometry. The fundamental input data will be common for all three systems; a = 0.010m, (A/a) = 1, (kN/a) = 30 and (kM/a) = 60. Through them it is valid that kM = 2 kN and this is condition (38).

For the disc electrodes I obtain the following data: (2m/a)N = 4.7065 and (2m/a)M = 9.5145. If a = 0.010m, there will be it that AN = 0.024m and AM = 0.048m.

If I have the diamond electrodes I shall obtain the following results: (2m/a)N = 4.6410 and (2m/a)M = 9.4805. In case that a = 0.010m, I shall get that AN = 0.023m and AM = 0.047m.

For the square electrodes I receive these data: (2m/a)N = 2.7065 and (2m/a)M = 7.5145. For a = 0.010m it is valid that AN = 0.014m and AM = 0.038m.

All projected systems ought to have minimal deviation between the micro-normal and the micro-inverse, if they are in electrically-homogeneous surroundings. They are those systems depicted in fig.1.

You can put a question how it is when it holds that (A/a) ≠ 1. The most interesting relationships you find out in case of the diamond electrodes. I shall apply the same input data that a = 0.010m, (kN/a) = 30 and (kM/a) = 60 and I change the magnification factors: (A/a) = 2 and (A/a) = 0.5.

For (A/a) = 2 I shall get that (2m/a)N = 3.0390 and (2m/a)M =7.9645. If a = 0.010m the output data will be like this: AN = 0.015m and AM = 0.040m.

In case that (A/a) = 0.5 I obtain that (2m/a)N = 5.3965 and (2m/a)M = 10.2130. In such case when a = 0.010m I shall get that AN = 0.027m and AM = 0.051m.

Symmetrical and asymmetrical systems – evaluation of their horizontal radius

The volume of the electric field is defined like sphere having its centre in the current electrode A. Its radius is presented as the spacing of tool. That is a distance, for the micro-normal it is abscissa AM, for the micro-inverse it is abscissa AO. The last mentioned abscissa is expressed like this:

$$\underline{AO} = 0.5\times\left(\underline{AM} + \underline{AN}\right) \tag{41}$$

The electrode array can be symmetrical or asymmetrical. It is given by position of electrode N. The symmetrical array has usually bigger radius; for the asymmetrical array the radius can be lower. It depends on the micro-normal. In the contrary to this fact there exists definition of the eccentricity factor characterizing each of arrays.

If you borrow from the theory of conics definition of eccentricity, you will be allowed to apply this factor in an adjusted form for classification of the electrode arrays.

$$\varepsilon = \sqrt{1-\left(\frac{\underline{AN}}{\underline{AM}}\right)^2} - \frac{\sqrt{3}}{2} = \sqrt{2\times\left(\frac{\underline{AO}}{\underline{AM}}\right)\times\left(\frac{\underline{MN}}{\underline{AM}}\right)} - \frac{\sqrt{3}}{2}, \tag{42}$$

where ε = factor of eccentricity according to electrode N,
AO = spacing of the micro-inverse, and
MN = base of the micro-inverse.

If you are going to analyze this formula, you will result in the following facts:

1. For AN = 0.5× AM it holds that ε = 0 and that means that the electrode array is symmetrical.
2. For AN < 0.5× AM and on condition that AN → 0 it holds that ε = +0.134 and the electrode array has positive asymmetry.
3. For AN > 0.5× AM and on condition that AN → AM it holds that ε = -0.866 and the electrode array has negative asymmetry.

All arrays having been projected before you find out in tab.1 there. This table provides good informative ability about projected arrays.

Analysis of various electrode arrays of Microlog

The derived formulas of Microlog are expressed generally like function k/a = f (2m/a, 2n/a, A/a). The formulas for the classical axis array have reducing condition that (2n/a) = 0. Therefore the depicted plots use function k/a = f (2m/a, A/a).

For all axis arrays it is possible to write that k/a = f (2r/a, A/a) where symbol r presents distance between centres of the current and normal electrodes in any arbitrary direction.

Possibility you can count the geometric constant of the micro-normal has great significance for creation of various electrode arrays consisting of three or four electrodes on the pad. Fig.5 depicts available arrays of the three-electrode configuration. There are these: a. perpendicular one, b. horizontal axis one and c. classical axis one.

Fig. 6. presents arrays of the disc electrodes having four ones. There are presented some of the possible arrays: a. perpendicular one, b. axis one and c. Werner one. It ought to be emphasized these three arrays are only samples of big amount of available variances. And moreover, I have to add that not every array is convenient. I should like to remark that the derived formulas are exactly made for configurations depicted on fig. 5. and 6.

For the four-electrode array you can exactly enumerate four partial constants remarked kAM, kAN, kBM and kBN, whereas, the three-electrode array has only two constants: kAM and kAN. Thanks to well-known principles holding for the point electrodes you receive final constants for two/one potentials of the micro-normal and for one gradient of the micro-inverse. The formulas are in agreement to depiction in fig. 5. and 6.

For the three-electrode array in arbitrary position on the plane if I suppose that partial geometric constants kAM and kAN are vectors there holds this formula:

$$k_g = \begin{cases} \left[\left(k_{AM}^{-1}\right)^2 + \left(k_{AN}^{-1}\right)^2 - 2\times \left(k_{AM}^{-1}\right) \times \left(k_{AN}^{-1}\right) \times \cos\left(\alpha_M - \alpha_N\right) \right]^{-\frac{1}{2}} \ldots \text{for } 0 \le \left(\alpha_M - \alpha_N\right) \le \frac{\pi}{2} \\ \left[\left(k_{AM}^{-1}\right)^2 + \left(k_{AN}^{-1}\right)^2 + 2\times \left(k_{AM}^{-1}\right) \times \left(k_{AN}^{-1}\right) \times \cos\left(\alpha_M - \alpha_N\right) \right]^{-\frac{1}{2}} \ldots \text{for } \frac{\pi}{2} \le \left(\alpha_M - \alpha_N\right) \le \pi . \end{cases} \quad (43)$$

where (αM – αN) = the angle having its vertex in the centre of the current electrode.

If you analyze this formula, you will state that there exist three possible events.

1. For (αM – αN) = 0 it holds that cos (αM – αN) = +1.In such case it is valid that:

$$k_g = \left(k_{AN}^{-1} - k_{AM}^{-1} \right)^{-1}. \quad (44)$$

2. For (αM – αN) = π/2 it holds that cos (αM – αN) = 0. Then you will get this formula:

$$k_g = \frac{1}{\sqrt{\left(k_{AN}^{-1}\right)^2 + \left(k_{AM}^{-1}\right)^2}} . \quad (45)$$

3. For (αM – αN) = π it holds that cos (αM – αN) = -1.For that event it holds that:

$$k_g = \left(k_{AN}^{-1} - k_{AM}^{-1} \right)^{-1}.$$

At first you should suppose that constants are unequal, i.e., it holds that kAM ≠ kAN. All axis arrays are those which have (αM – αN) = 0 and (αM – αN) = π. The first presents that electrodes M and N are both on identical side with regard to electrode A; the second is that each of electrodes M and N are on opposite side after electrode A.

From this point we can classify the single array depicted on fig.5 and fig.6. For the classical three-electrode array there are these formulas:

$$k_{pM} = k_{AM} .$$

$$k_g = \left(k_{AN}^{-1} - k_{AM}^{-1} \right)^{-1}.$$

For the four-electrode axis array remarked as the axis one you receive the following formulas:

$$k_{pM} = \left(k_{BM}^{-1} - k_{AM}^{-1} \right)^{-1} \tag{46}$$

$$k_{pN} = \left(k_{BN}^{-1} - k_{AN}^{-1} \right)^{-1} \text{ and} \tag{47}$$

$$k_g = \left[\left(k_{BN}^{-1} - k_{BM}^{-1} \right) - \left(k_{AN}^{-1} - k_{AM}^{-1} \right) \right]^{-1} \tag{48}$$

The next axis array is Werner array. For this there hold the following formulas:

$$k_{pN} = \left(k_{AN}^{-1} - k_{BN}^{-1} \right)^{-1} \tag{49}$$

$$k_{pM} = \left(k_{BM}^{-1} - k_{AM}^{-1} \right)^{-1} \text{ and}$$

$$k_g = \left[\left(k_{AN}^{-1} - k_{BN}^{-1} \right) - \left(k_{BM}^{-1} - k_{AM}^{-1} \right) \right]^{-1}. \tag{50}$$

It is clear that for B→ ∞ you attain the classical axis array, when

$k_{BN}^{-1} = 0$ and $k_{BM}^{-1} = 0$.

There remain arrays having both constant equal; it holds that kAM = kAN . In such case it is three-electrode perpendicular array and you obtain formula as follows:

$$k_g = \begin{cases} \dfrac{k_{AM}}{2 \times \sin\left(\dfrac{\alpha_M - \alpha_N}{2} \right)} & \ldots \text{for } 0 \langle \left(\alpha_M - \alpha_N \right) \le \dfrac{\pi}{2} \\ \dfrac{k_{AM}}{2 \times \cos\left(\dfrac{\alpha_M - \alpha_N}{2} \right)} & \ldots \text{for } \dfrac{\pi}{2} \le \left(\alpha_M - \alpha_N \right) \le \pi . \end{cases} \tag{51}$$

If you analyze this formula you will get the following results:

For (αM – αN) = 0 it holds that sin [(αM – αN)/2] = 0 and kg = ∞. This presents identity of electrodes M and N; such case cannot be.

For (αM – αN) = π/2 it holds that sin [(αM – αN)/2] = √2/2. Then it holds that:

$$k_g = \frac{\sqrt{2}}{2} \times k_{AM} . \tag{52}$$

For (αM – αN) = π it holds that cos [(αM – αN)/2] = 0 and kg = ∞. It is case when between electrodes M and N there is zero voltage.

The three-electrode array remarked as the horizontal axis array belong to the last named. For this is kAM = kAN and (αM – αN) = π. This array is not convenient for usual registration, because if kg = ∞ then formula (35) says that ΔU = 0, permanently. But for registration of vertical micro-inhomogeneities being between electrodes M and N it may be used.

The last array is the four-electrode one remarked as the perpendicular array. For this there are the following formulas:

$$k_g = \begin{cases} +\dfrac{1}{2}\times\left\{\dfrac{k_{BM}}{\sin\left(\dfrac{\alpha_{BM}-\alpha_{BN}}{2}\right)} - \dfrac{k_{AM}}{\sin\left(\dfrac{\alpha_{AM}-\alpha_{AN}}{2}\right)}\right\} & \ldots \text{for } 0 < (\alpha_{AM}-\alpha_{AN}) \le \dfrac{\pi}{2},\ 0 < (\alpha_{BM}-\alpha_{BN}) \le \dfrac{\pi}{2} \\ -\dfrac{1}{2}\times\left\{\dfrac{k_{BM}}{\cos\left(\dfrac{\alpha_{BM}-\alpha_{BN}}{2}\right)} - \dfrac{k_{AM}}{\cos\left(\dfrac{\alpha_{AM}-\alpha_{AN}}{2}\right)}\right\} & \ldots \text{for } \dfrac{\pi}{2} \le (\alpha_{AM}-\alpha_{AN}) < \pi,\ \dfrac{\pi}{2} \le (\alpha_{BM}-\alpha_{BN}) < \pi \end{cases}$$

(53)

$$k_p = \left(k_{AM}^{-1} - k_{BM}^{-1}\right)^{-1} = \left(k_{AN}^{-1} - k_{BN}^{-1}\right)^{-1}.$$

(54)

The fact you are able to count exactly constants of Microlog for arbitrary array significantly extends the construction domain of tools having different convenient arrays of electrodes. Here is opened a wide way of experiments for new types of Microlog.

I attempted by mathematical modeling to find out the root principle of Microlog behavior tending to new knowledge, because there is always actual Latin phrase: Causarum cognitio cognitionem eventorum facit.

Conclusions

In accordance to made up analysis I present these conclusions:

Different shape geometry of electrodes affects their electric field and this is why of various shapes of relations having various magnification factors. Nonlinearity of those relations depends on magnification factor there where short distances between electrodes are. In the domain of long distances, when it holds that (2m/a) >> 1 and (2n/a) >> 1, there act only translation factors in horizontal and vertical directions. All relations incline to the basic relationship characterizing the point electrodes.

In the electrically-homogeneous surroundings there is registered identical resistivity for both the micro-normal and the micro-inverse. It is insured with condition that kM = 2× kN. Thanks to it there are valid next partial conditions: ΔU = UM and kp = kg = kM. This condition holds for both linear and nonlinear relations.

The electrode arrays can be symmetrical or asymmetrical. For the symmetrical ones it holds that AN = MN, whereas, the asymmetrical ones are characterized either with condition that AN > MN, negative asymmetry, or with condition that AN < MN, positive asymmetry.

Thanks to derived function k/a = f (2m/a, 2n/a, A/a) it is possible exactly to compute geometric constants of the micro-normal and the micro-inverse in arbitrary place of plane for arbitrary electrode array. This opens new ways for new construction of unconventional electrode arrays.

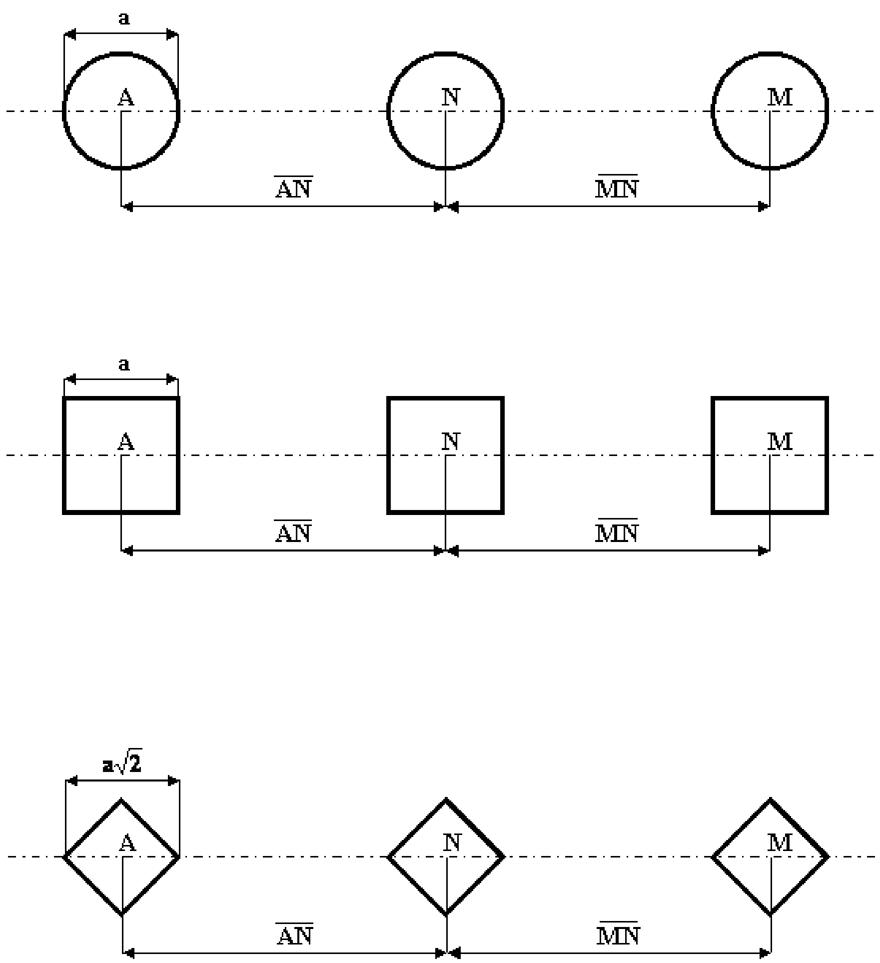

Fig. 1 The various form of electrodes for classical Mikrolog

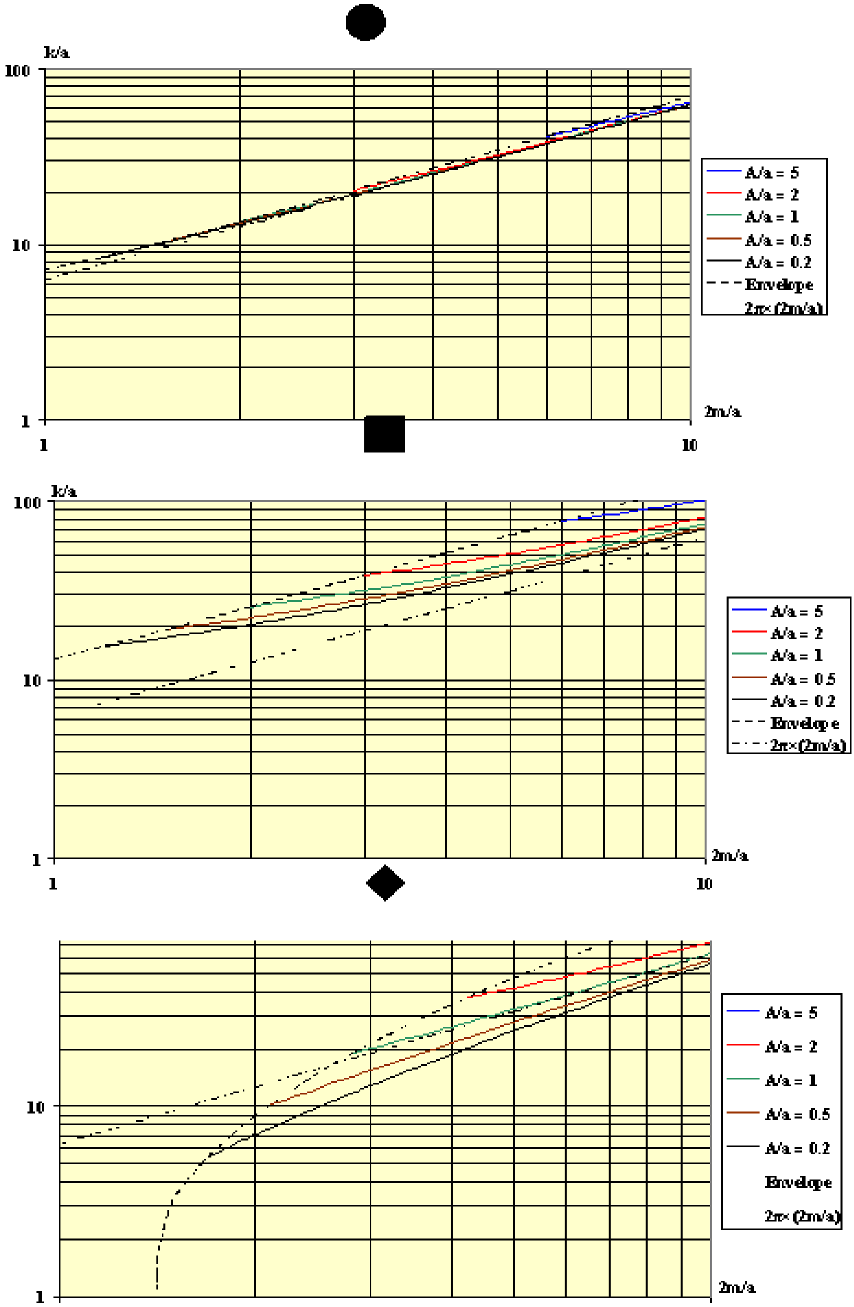

Fig. 2 Comparison of various form electrodes like function k/a = f (2m/a, A/a)

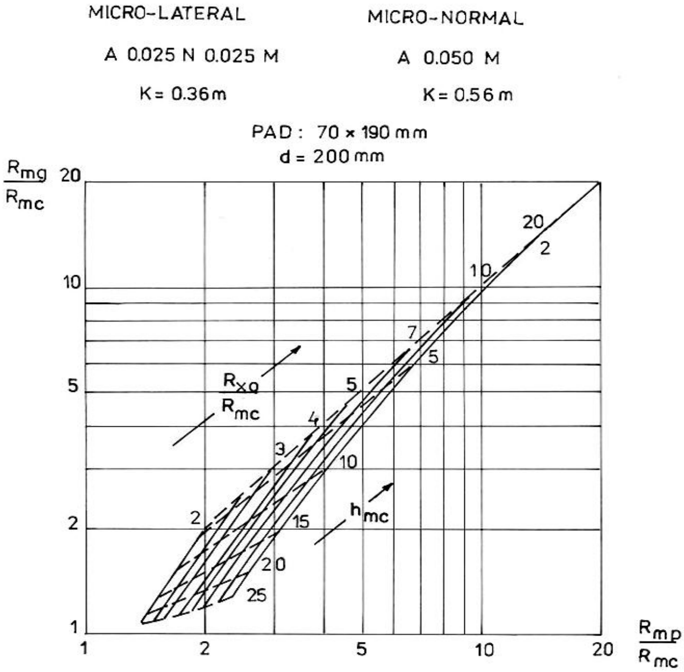

Fig.3 Correction charts for Russian Microlog for pad 70 × 190mm

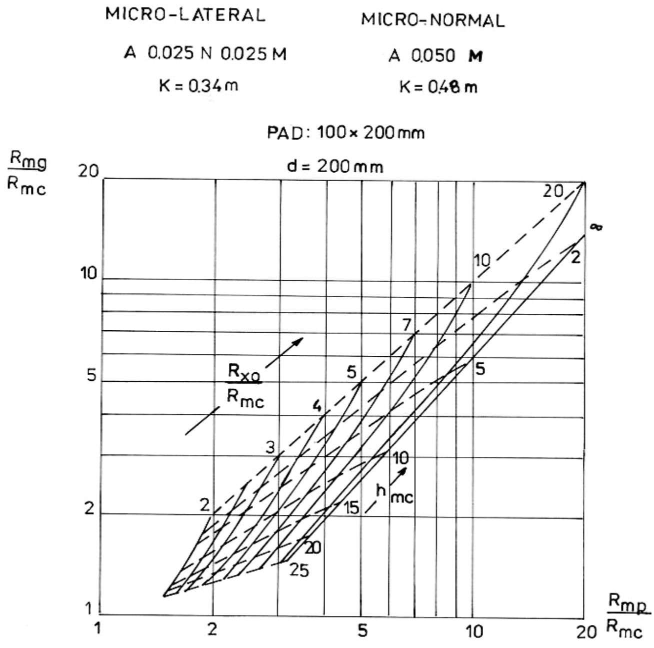

Fig. 4 Correction charts for Russian Microlog for pad 100 × 200mm

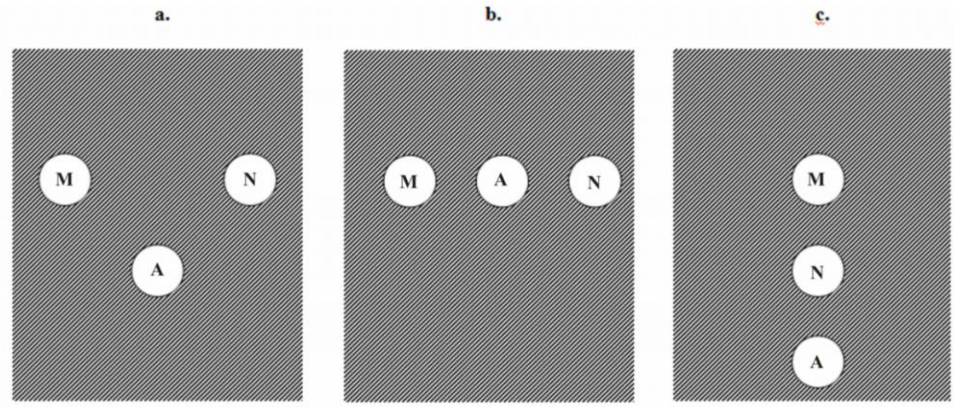

Fig.5 Some of possible arrays for the three-electrode system of Microlog

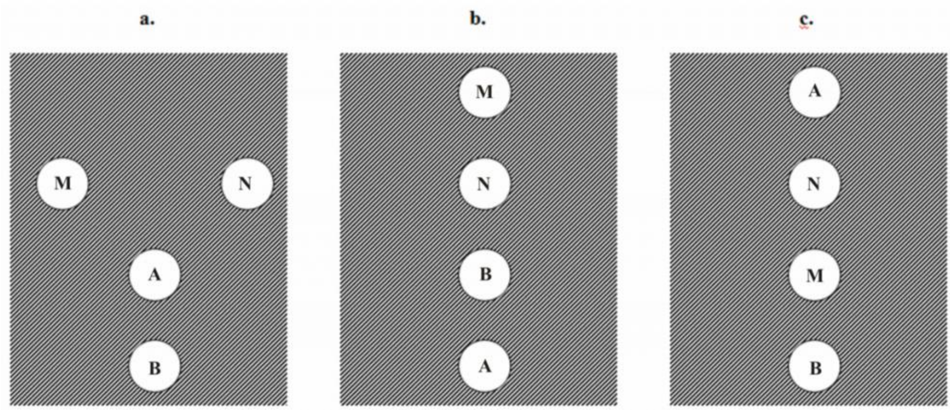

Fig. 6 Some of possible arrays for the four-electrode system of Microlog

Tab. 1 Examples of different shape geometry and their classification

Shape geometry	(A/a)	AN [mm]	AM [mm]	MN [mm]	AO [mm]	(AO/AM)	(MN/AM)	ε
Disc	1	24	48	24	36	0.750	0.500	0.000
Square	1	14	38	24	26	0.684	0.632	0.064
Diamond	1	23	47	24	35	0.745	0.511	0.006
	2	15	40	25	27.5	0.688	0.625	0.061
,	0.5	27	51	24	39	0.765	0.471	-0.018